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Remarks on confinement driven by axion-like particles in Yang–Mills theories

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Abstract
Features of screening and confinement are studied for a non-Abelian gauge theory with a mixture of pseudoscalar and scalar couplings, in the case where a constant chromoelectric, or chromomagnetic, strength expectation value is present. Our discussion is carried out using the gauge-invariant but path-dependent variables formalism. We explicitly show that the static potential profile is the sum of a Yukawa and a linear potential, leading to the confinement of static probe charges. Interestingly, similar results have been obtained in the context of gluodynamics in curved spacetime. For only pseudoscalar coupling, the results are radically different.

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One of the most challenging, and still open, problems in high energy theoretical physics is the quantitative description of confinement in quantum chromodynamics (QCD). Recent advances in string theory provide a promising new framework to face the non-perturbative features of Yang–Mills theories (for a recent review, see [1]).
However, phenomenological models still represent a key tool for understanding confinement physics. In this context we recall the illustrative scenario of dual superconductivity [2], where it is conjectured that the QCD vacuum behaves as a dual-type II superconductor. More precisely, because of the condensation of magnetic monopoles, the chromoelectric field acting between $q\overline{q}$ pairs is squeezed into strings (flux tubes), and the nonvanishing string tension represents the proportionality constant in the linear potential. Lattice calculations have confirmed this picture by showing the formation of tubes of gluonic fields connecting colored charges [3].

With these considerations in mind, in a previous work [4], we have studied an effective non-Abelian gauge theory where a Cornell-like profile is obtained in the presence of a nontrivial constant expectation value for the gauge field strength $\langle F^{a}_{\mu\nu}\rangle$ coupled to a light pseudoscalar boson field φ ('axion'). In fact, this theory experiences mass generation due to the breaking of rotational invariance induced by the classical background configuration of the gauge field strength, and in the case of a constant chromoelectric field strength expectation value the static potential remains Coulombic. Nevertheless, this picture drastically changes in the case of a constant chromomagnetic field strength expectation value. In effect, the potential energy is the sum of a Coulomb and a linear potential, leading to the confinement of static charges. It should be noted that the magnetic character of the field strength expectation value needed to obtain confinement is in agreement with the current chromomagnetic picture of the QCD vacuum [5]. Incidentally, the above static potential profile is analogous to that encountered in Yang–Mills theory with spontaneous symmetry breaking of scale symmetry [6]. This then implies that although the constraint structure of the two models is quite different, the physical content is identical. As a result, our study has provided a new kind of 'duality' between effective non-Abelian theories.

On the other hand, in recent times the coupling of axion-like particles with photons in the presence of an external background electromagnetic field and its physical consequences have been the object of intensive investigations [7–19], after recent results of the PVLAS collaboration [20]. Let us also mention here that these effects can be qualitatively understood by the existence of light pseudoscalar bosons ϕ ('axions'), with a coupling to two photons. In this context, it was suggested in [22] that the spin-zero particle describing the PVLAS results could be one of no definite parity. The reason is that within the low energy regime used by PVLAS the spin-zero particle could well be one of no definite parity, that is, a mixture of pseudoscalar and scalar. Certainly, if the PVLAS results are supported by further experimental data, it would signal new physics containing very light bosons [23]. Given its relevance, it is of interest to understand better the impact of spin-zero particle–gluon interactions on a physical observable. Seem from such a perspective, the present work is an extension of our previous studies started in [21] and continued in [4]. To do this, we will work out the static potential for a theory which includes scalar and pseudoscalar particles coupled to a non-Abelian gauge field using the gauge-invariant but path-dependent variables formalism. Our treatment is fully non-perturbative for the spin-zero field. As a result, we obtain that the potential energy is the sum of a Yukawa and a linear potential, leading to the confinement of static charges, which clearly shows the key role played by the scalar particle in transforming the Coulombic potential into the Yukawa one. This may be contrasted with the role played by the noncommutative space in transforming the Yukawa potential into the Coulombic one, in the context of noncommutative axionic electrodynamics [24]. Interestingly enough, the above static potential profile is analogous to that encountered in gluodynamics in curved spacetime [25]. Therefore, the above result reveals a new equivalence between effective non-Abelian theories, in spite of the fact that they have different constraint structures. Accordingly, the gauge-invariant but path-dependent variables formalism offers an alternative view in which some features of effective non-Abelian gauge theories become more transparent.

We shall now discuss the interaction energy between static point-like sources for the model under consideration. To this end we will compute the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle H\rangle_\Phi$. The starting point is the Lagrangian density:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{\lambda_+}{4}\varphi F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda_-}{4}\varphi\tilde{F}^a_{\mu\nu}F^{a\mu\nu}, \quad (1)$$

where m is the mass for the spin-zero particle. Here, $A_\mu(x) = A^a_\mu(x)T^a$, where T^a is a Hermitian representation of the semi-simple and compact gauge group, and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$, with f^{abc} as the structure constants of the group, whereas λ_+ and λ_- are coupling constants for scalar and pseudoscalar particles, respectively.

The Lagrangian (1) provides an *effective* description of axion-like particles interacting with chromoelectric and chromomagnetic fields. Thus, from a phenomenological point of view, m^2 and $\lambda_\pm$ were put 'by hand' and not related in any simple way to the Yang–Mills coupling constant g and energy scale $\Lambda_{\rm QCD}$.

As we have indicated in [21], to compute the interaction energy we need to carry out the integration over the φ-field. Once this is done, we arrive at the following effective theory for the gauge fields:

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda_+^2}{32}F^a_{\mu\nu}F^{a\mu\nu}\frac{1}{\Delta+m^2}F^b_{\mu\nu}F^{b\mu\nu} + \frac{\lambda_-^2}{32}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\frac{1}{\Delta+m^2}\tilde{F}^{b\mu\nu}F^b_{\mu\nu} + \frac{\lambda_+\lambda_-}{16}F^a_{\mu\nu}F^{a\mu\nu}\frac{1}{\Delta+m^2}\tilde{F}^{b\mu\nu}F^b_{\mu\nu}. \qquad (2)$$

Next, after splitting $F^a_{\mu\nu}$ in the sum of a classical background $\langle F^a_{\mu\nu}\rangle$ and a small fluctuation $f^a_{\mu\nu}$, the corresponding Lagrangian density becomes

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}f^a_{\mu\nu}\left[1-\frac{3\lambda_+^2 v^{c\lambda\rho}v^c_{\lambda\rho}}{\Delta+m^2}\right]f^{a\mu\nu} + \frac{\lambda_-^2}{32}v^{a\alpha\beta}f^a_{\alpha\beta}\frac{1}{\Delta+m^2}v^{b\gamma\delta}f^b_{\gamma\delta}. \qquad (3)$$

Here we have simplified our notation by setting $\varepsilon^{\mu\nu\alpha\beta}\langle F^a_{\mu\nu}\rangle \equiv v^{a\alpha\beta}$ and $\varepsilon^{\rho\sigma\gamma\delta}\langle F^b_{\rho\sigma}\rangle \equiv v^{b\gamma\delta}$.

We remark that the new feature of the present model is the non-trivial presence of the term proportional to λ_+^2. This point motivates us to study the role of the scalar field on a physical observable.

We now turn our attention to the calculation of the interaction energy in the $v^{a0i}\neq 0$ and $v^{aij}=0$ case (referred to as the electric one in what follows). In such a case, the Lagrangian (3) reads

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}f^a_{\mu\nu}\left(1+\frac{6\lambda_+^2(v^c)^2}{\Delta+m^2}\right)f^{a\mu\nu} + v^{ai0}f^a_{i0}\frac{\lambda_-^2/8}{\Delta+m^2}v^{bk0}f^0_{k0}. \qquad (4)$$

The canonical Hamiltonian can be worked as usual and is given by

$$H_C = \int d^3x\left[\Pi^{ai}\left(\partial_i A^a_0 + g f^{abc}A^c_0 A^b_i\right) + \frac{1}{2}B^{ai}\left(1+\frac{6\lambda_+^2(v^c)^2}{\Delta+m^2}\right)B^{ai}\right] + \int d^3x\left[\frac{1}{2}\Pi^{ai}\frac{\Delta+m^2}{\Delta+M^2}\Pi^{ai} - \frac{\lambda_-^2}{8}(v^{ai}\Pi^{ai})\frac{1}{\Delta+\mathcal{M}^2}(v^{bi}\Pi^{bi})\right], \qquad (5)$$

where B^{ai} is the chromomagnetic field, $M^2\equiv m^2+6\lambda_+^2(v^a)^2$, and $\mathcal{M}^2\equiv M^2+\lambda_-^2/4(v^a) = m^2+\left[6\lambda_+^2+\lambda_-^2/4\right](v^a)^2$.

By proceeding in the same way as in [4], we obtain the static potential for two opposite charges located at $\mathbf{0}$ and $\mathbf{y}$:

$$V = -\frac{g^2}{4\pi}C_F\frac{e^{-ML}}{L} + g^2(\xi+g^2\xi')L, \qquad (6)$$

where

$$\xi \equiv \frac{1}{2\pi}\left[\frac{m^2}{4}C_F\ln\left(1+\frac{\overline{\Lambda}^2}{M^2}\right) - \frac{\lambda_-^2}{16}{\rm tr}(v^{ai}T^a v^{bi}T^b)\ln\left(1+\frac{\tilde{\Lambda}^2}{M^2}\right)\right] \qquad (7)$$

and

$$\xi' \equiv {\rm tr}(f^{abc}f^{adc}T^bT^d)\left[\frac{1}{8\pi}\left(\Lambda^2 - M^2\ln\left(1+\frac{\Lambda^2}{M^2}\right)\right) + \frac{m^2}{4}\ln\left(1+\frac{\overline{\Lambda}^2}{M^2}\right)\right] - \frac{\lambda_-^2}{16}{\rm tr}(v^{pi}f^{pbc}T^b v^{qi}f^{qdc}T^d)\ln\left(1+\frac{\tilde{\Lambda}^2}{\mathcal{M}^2}\right), \qquad (8)$$

where $\overline{\Lambda}$ and $\tilde{\Lambda}$ are cutoffs.

Expression (6) immediately shows both expected and unexpected features of the model. The linear confining piece was expected from our previous study [4]. The novel feature is the Yukawa piece. In fact, the above result clearly reveals the key role played by the scalar particle (λ_+ term) in transforming the Coulombic potential into the Yukawa one. As already expressed, a similar form of interaction potential has been reported earlier in the context of gluodynamics in curved spacetime [25]. Also, a common feature of these models is that the rotational symmetry is restored in the resulting interaction energy.

Now we focus on the case $v^{a0i} = 0$ and $v^{aij} \neq 0$, which we refer to as the magnetic one in what follows. Thus, we obtain from (3)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} f^a_{\mu\nu} \left(1 + \frac{6\lambda_+^2 (v^c)^2}{\Delta + m^2}\right) f^{a\mu\nu} + v^{aij} f^a_{ij} \frac{\lambda_-^2/32}{\Delta + m^2} v^{bkl} f^0_{kl}, \tag{9}$$

where $\mu, \nu = 0, 1, 2, 3$ and $i, j, k, l = 1, 2, 3$. Here again, the quantization is carried out using Dirac's procedure. The canonically conjugate momenta, as obtained from (9), are

$$\Pi^{ao} = 0, \tag{10}$$

$$\Pi^a_i = D^{ab}_{ij} f^b_{j0}, \tag{11}$$

$$D^{ab}_{ij} \equiv \delta^{ab} \left(1 + \frac{6\lambda_+^2 (v^c)^2}{\Delta + m^2}\right) \delta_{ij}. \tag{12}$$

The explicit form of the chromoelectric field turns out to be

$$E^a_i = \frac{\Delta + m^2}{\Delta + M^2} \Pi^a_i, \tag{13}$$

where $M^2 \equiv m^2 + 6\lambda_+^2 (v^a)^2$. This leads us to the canonical Hamiltonian

$$H_{\rm C} = \int {\rm d}^3x \left[\Pi^{ai} \left(\partial_i A^a_0 + g f^{abc} A^c_0 A^b_i\right) + \frac{1}{2} B^{ai} \left(1 + \frac{6\lambda_+^2 (v^c)^2}{\Delta + m^2}\right) B^{ai}\right] + \int {\rm d}^3x \left[\frac{1}{2} \Pi^{ai} \frac{\Delta + m^2}{\Delta + M^2} \Pi^{ai}\right], \tag{14}$$

where B^{ai} is the chromomagnetic field.

We skip all the technical details and refer to [4] for them. The static potential turns out to be

$$V = -\frac{g^2}{4\pi} C_F \frac{{\rm e}^{-ML}}{L} + g^2 (\xi + g^2 \xi') L, \tag{15}$$

where

$$\xi = \frac{m^2}{8\pi} C_F \ln\left(1 + \frac{\overline{\Lambda}^2}{M^2}\right) \tag{16}$$

and

$$\xi' = \frac{1}{4} \left[\frac{C_F C_A \sigma}{2\pi^2} + m^2 {\rm tr}(f^{abc} T^b f^{adc} T^d) \ln\left(1 + \frac{\overline{\Lambda}^2}{M^2}\right)\right]. \tag{17}$$

Here, in contrast to our previous analysis [4], unexpected features are found. Interestingly, it is observed that the introduction of the λ_+ term induces a Yukawa piece plus a linear confining piece. Note that in the $\lambda_+ = 0$ case, the static potential remains Coulombic. Again, the rotational symmetry is restored in the resulting interaction energy despite that the chromoelectric external field breaks this symmetry.

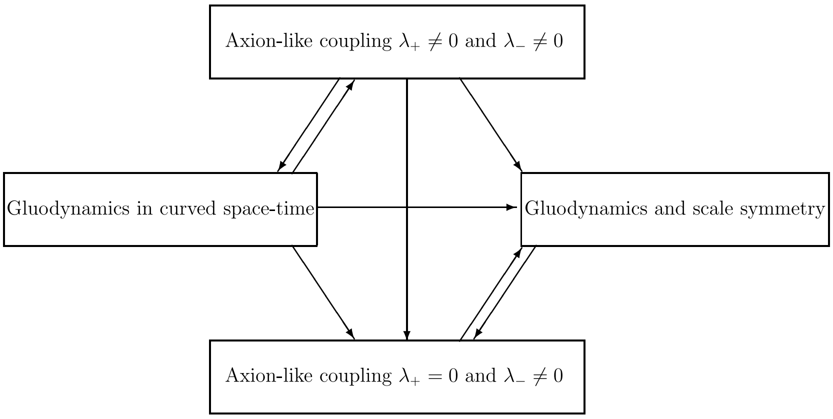

Figure 1. 'Duality web' between different Yang–Mills–Axiom models.

The presence of mass terms, coming from a non-vanishing background value for $F^a_{\mu\nu}$, suggests a possible analogy with the model introduced in [27], where the gluon mass is produced through a non-vanishing vacuum expectation value of the composite operator A^2_μ. However, our approach is substantially different: the distinctive feature of our method is to define the interaction potential between test charges in a manifestly gauge-invariant way. On the other hand, the non-vanishing vacuum expectation value $\langle A^2_\mu \rangle$ in [27] is gauge dependent. The authors claim that under some circumstances this quantity can be given a gauge-invariant meaning, and we trust them, but a direct comparison with our approach is very hard, at this level. A second difference, which is worth to remark, is different kinds of 'energies' which are considered. The main purpose of [27] is to resolve the instability of the Saviddy vacuum. Accordingly, the authors compute one-loop effective potential, i.e. vacuum energy density, in the presence of a background chromomagnetic field and a condensate for A^2_μ. On our side, we determine the interaction energy between static charges, which is the static potential energy. Comparison with [27] would require the calculation of the one-loop vacuum energy density for our model, but this is a problem which is interesting by itself and cannot be fully investigated in this short paper.

Let us put our work in its proper perspective. This paper is a sequel to [4, 21], where we have exploited a crucial point for understanding the physical content of gauge theories, that is, the identification of field degrees of freedom with observable quantities. Our analysis reveals both expected and unexpected features of the model studied. It was shown that the static potential profile is the sum of a Yukawa and a linear potential, leading to the confinement of static probe charges. This result is obtained for both external chromomagnetic and chromoelectric strength expectation values. This may be contrasted with our previous study [4], where only a pseudoscalar coupling was considered. Also, the above analysis has showed the key role played by the scalar particle in transforming the Coulombic potential into the Yukawa one. Interestingly, similar results have been obtained in the context of gluodynamics in curved spacetime [25]. An important consequence of this is that, although the constraint structure of the two models is quite different, the physical content is identical. This means that our study has provided a new kind of 'duality' between effective non-Abelian theories which is summarized in figure 1.

We conclude noting that our results agree with the dilaton coupled to the gauge fields mechanism [26]. However, although both approaches lead to confinement, the above analysis reveals that the mechanism of obtaining a linear potential is quite different. As already mentioned, in this work we have exploited the similarity between the tree level mechanism that leads to confinement here and the nonperturbative mechanism (caused by quantum effects) which gives confinement in QCD in curved spacetime.

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